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## **Non-parametric Tests of Consumer Behavior in Transition Economies**

by

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and

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## **ABSTRACT**

We re-examine the notion of consumer rationality in transition economies by modifying the conventional non-parametric tests of the axioms of revealed preference. To reflect the unique characteristics of transition economies, shortages are allowed in the state market, while not in the private market.

Because of the way the modified tests are constructed, data for prices in both the state and private markets are needed, as are data on purchases from state markets. Through some convenient algebraic substitutions, the tests do not require data on actual quantities purchased in the private markets. This feature of the modified tests is especially important, given the near impossibility of gaining access to data on sales or purchases from private markets in transition economies.

Using limited aggregate consumption data for four important food items in five cities of the former Soviet Union, we clearly demonstrate that conventional non-parametric tests can lead to an erroneous rejection of the rationality hypothesis of consumer behavior for economies in transition.

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# **Non-parametric Tests of Consumer Behavior in Transition Economies**

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## ***Introduction***

One of the major challenges for economists since the collapse of the Eastern Bloc is to understand the changes in consumer behavior as centrally planned economies (CPE) make the transition to more market-oriented economies. During the transition period, many of these economies have experienced severe shortages, stagflation, and the rapid development of alternative markets. These disturbances cause dramatic changes in consumption opportunities, which in turn affect consumer behavior.

There is some limited empirical work suggesting that, during these transition period disturbances, consumer behavior may appear to be irrational and cannot be described as a result of utility maximization (Shananin, 1990; 1993). Consumers in emerging markets, however, face a number of constraints in addition to a traditional budget constraint. Among the most important constraints in former Eastern Bloc countries are related to product availability, time, and product quality. To analyze the rationality of the consumer behavior in emerging markets, we must grasp the importance of these constraints and consider them explicitly in theoretical and empirical analyses.

Under the fixed price system of the centrally planned economies, for example, consumers often fail to find goods they desire (Davis and Charemza, 1989). Shortages in the state supermarkets force consumers either to postpone their purchases or to revise

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initial demands and substitute alternative available goods. Hyperinflation, also a common characteristic of transition periods, encourages immediate spending and hoarding.

The long queues outside state shops even when the desired goods are available certainly have affected demand formation and consumption behavior during the transition period. Since wages in the private sector are adjusted for inflation almost weekly, and working hours are flexible, consumers engaged in private sector employment are unlikely to spend much time waiting to make purchases in state shops, if an alternative exists.

The rapid expansion of the private sector and the absence of salary inflation adjustments in the state sector have led to large income disparities. What was once a relatively homogeneous population is quickly divided into different income groups, and the gap between them widens rapidly. These huge income gaps have segmented consumer goods markets and, in turn, have led to the development and rapid expansion of private markets as alternatives to state shops.

Product quality also may be higher in private markets, because producers want the higher prices for their best products. With fixed prices in the state shops, there are no incentives for managers of state shops to invest in product development; nor is there any incentive for efficiency in the central distribution system supplying state shops. The inferior quality of many perishable commodities may result from late deliveries due to the malfunctioning of the state transportation system.

There are two kinds of buyers in the state shops: households and firms. In making purchases from state shops, households operate under effective budget constraints, and they cannot spend more than they have at their disposal. In contrast, even though state

firms are given budgets, they normally purchase as much as they wish. In this case, both the seller and the buyer are under state control, and credit agreements between state enterprises are usually not enforced. The situation is again different in alternative markets where all buyers must operate within their budgets because all the sellers are private, profit maximizing agents.

During times of shortage in the state shops, private supply is increased in order to absorb the existing excess demand. However, consumers increase their purchases expecting future shortages. As a result, shortages persist, a phenomenon that has its origins in the 1930's, where demands for consumer goods often went unmet because of deliberate CPE state policies to stimulate industry, the main sector of the socialist economy.

Shortages of goods have been studied within the context of a model of a shortage economy (Clower, 1965; Kornai, 1971, 1980) and within the context of disequilibrium models (e.g. the papers in Davis and Charemza, 1989), and others have focused on substitution, forced saving, and queues (e.g., Mekhtiev, 1997). No one has examined the effects of the existence of alternative markets on consumer behavior.

The purpose of this paper is to provide a method for investigating the economic rationality of consumer behavior in transition economies. To do so, the question of consumer rationality is re-examined within the context of there being an emerging private market. Put differently, we wish to know if explicit recognition of the existence of both state and private markets in the examination of consumption data from state markets will point to the rationality of consumer behavior when more conventional analysis of the data would suggest otherwise.

We examine this issue within the context of a non-parametric approach to consumer behavior. To recognize the existence of two markets in the test of the rationality hypothesis, we modify the non-parametric tests of the axioms of revealed preference, which were originally developed by Afriat (1967) and Varian (1982; 1983). To reflect the unique characteristics of transition economies, shortages are allowed in one market, while private markets are assumed to clear. Because consumption data from private markets are generally not available, the real challenge is to construct modified tests which rely only on data on consumption from state shops and prices from both state and private markets. The method is demonstrated using aggregate consumption data from state markets for 1992-94 for four important food items in five cities of the former Soviet Union.

To help describe how the existing non-parametric tests must be modified, we proceed with a brief reiteration of the conventional well known Weak and Generalized Axioms of Revealed Preference (WARP, GARP).<sup>1</sup> We then characterize the two markets and develop the modified non-parametric tests. We present the results from an empirical application of the method and close with some final observations about their implications.

### ***The Conventional Non-parametric Tests***

The conventional non-parametric test of the hypothesis of consumer rationality checks to see if observed data are consistent with utility maximization through application of weak and generalized axioms of revealed preference (Varian, 1982). No

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<sup>1</sup> Driscoll and McGuirk (1997) modify these standard non-parametric tests to account for consumer perceptions of the fat content in investigating possible structural change in consumer demand for meat.

explicit assumptions about the functional form of the demand system are required. Only observed data on prices and consumption are needed to detect violations of the axioms.

Suppose for some number of time periods ( $t=1,...,T$ ), we have data on price vectors and consumption bundles,  $(P_t, X_t)$ . If there are  $N$  goods,  $X_t=(X_t^1,X_t^2,...,X_t^N)$  and  $P_t=(P_t^1,P_t^2,...,P_t^N)$ .

**Definition 1.** An observed consumption bundle  $X_i$  is *directly* revealed preferred to a bundle  $X_j$ , if  $P_i X_i \geq P_i X_j$ .

If the consumer chooses a bundle  $X_i$  at the prices  $P_i$  while bundle  $X_j$  is attainable at the same prices  $P_i$  with the same budget outlay, then bundle  $X_i$  is directly revealed preferred to bundle  $X_j$ . It is usually denoted  $X_i R X_j$ .

**Weak Axiom of Revealed Preference (WARP):** If bundle  $X_i$  is *directly* revealed preferred to bundle  $X_j$ ,  $X_i R X_j$ , then  $X_j$  cannot be revealed preferred to  $X_i$ , not  $X_j R X_i$ .

WARP, stated in terms of expenditures, is:

If  $X_i$  is directly revealed preferred to  $X_j$ , then this implies that  $X_j$  should have been obtainable for a given budget outlay:

$$(1) \quad P_i X_i \geq P_i X_j$$

If WARP is satisfied ( $X_i R X_j$  and not  $X_j R X_i$ ), we know

$$(2) \quad P_i X_i \geq P_i X_j \text{ and } P_j X_j < P_j X_i.$$

For the case of two goods, Figure 1 demonstrates situations in which WARP is satisfied and in which it is violated. At time period  $i$ , the ruling prices are  $P_i$ , and bundle  $X_i$  is chosen. The budget line at this time  $i$  passes through bundle  $X_i$ . When the consumption bundle  $X_j$  is chosen at the prices  $P_j$ , the budget line passes through bundle  $X_j$ . WARP is satisfied in this case as  $X_i$  was chosen while  $X_j$  was attainable and  $X_j$  was chosen while  $X_i$



was not attainable. At prices  $P_i$ , WARP would be violated if  $X_k$  is chosen instead of  $X_i$ . In this case,  $X_k$  is chosen while  $X_j$  is attainable but  $X_j$  is chosen while  $X_k$  is still attainable. This is a violation of WARP.

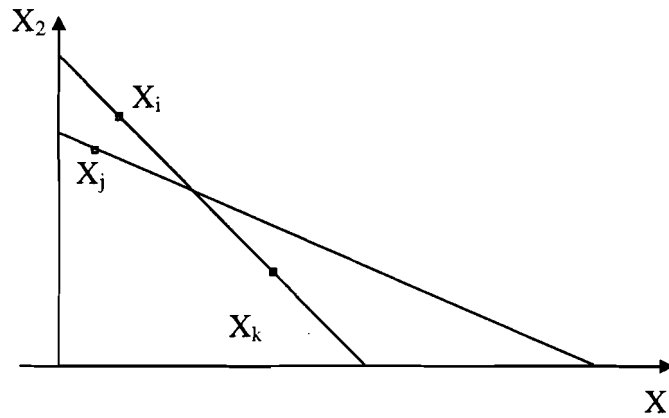


Figure 1: WARP

**Definition 2.** An observed consumption bundle  $X_i$  is revealed preferred to a bundle  $X_j$ , denote  $X_i P X_j$ , if there exist a sequence of bundles  $(X_k, X_m, \dots, X_r)$ , such that  $X_i R X_k, X_k R X_m, \dots, X_r R X_j$ .

**Generalized Axiom of Revealed Preference (GARP):** If  $X_i$  is revealed preferred to  $X_j$ ,  $X_i P X_j$ , then  $X_j$  cannot be revealed preferred to  $X_i$ , not  $X_j P X_i$ .

GARP checks every possible transitive closure, while WARP compares only two observed bundles. A violation of WARP leads to a violation of GARP, but not vice versa.

**Definition 3.** Utility function  $U(X)$  rationalizes the data  $(P_t, X_t)$ ,  $t=1, \dots, N$ , if  $U(X_t) \geq U(X)$  for all  $X$  such that  $P_t X_t \geq P_t X$ .

**Afriat's Theorem:** The following conditions are equivalent (Afriat, 1967):

- the data satisfy GARP;
- there exist numbers  $U^i, \lambda^i > 0$ ,  $i=1, \dots, N$ , such that  $U^i \leq U^j + \lambda^j P_j (X_i - X_j)$  for  $i, j=1, \dots, N$ ;

- there exists a non-satiated, continuous, concave, monotonic utility function that rationalizes the data.

If the data satisfy GARP, then this implies that consumer choices are consistent with the hypothesis of utility maximization behavior. If the data produce no cases of GARP violation, then there exist a demand system that satisfies the general properties of neoclassical demand theory.

### ***The Modified Non-Parametric Tests to Account for a Second Market***

*Behavior in the two markets.* During the transition period from centrally planned to market-oriented economies, alternative markets develop, and consumers begin to incorporate them into their purchasing decisions. Consumer demand formation can be characterized as follows. First, an initial demand is formed, and a shopping route is started to satisfy the initial demand. Second, a part of consumer's initial demand is realized at the state shops, as goods at the state shops are much cheaper than at alternative markets. Shortages of some goods at the state shops, however, are likely to prevent consumers from satisfying their *entire* initial demand. Since goods are always available at the alternative markets, the unsatisfied part of the initial demand *can* be purchased at the alternative market, but at higher prices. This is the key characteristic of the transition period, and it is in stark contrast to the pre-transition periods when goods are not available at the state shops and alternative markets, if they exist at all, have little or no capacity to accommodate the unsatisfied demand.

Given this nature of consumer demand formation during the transition period, the conventional non-parametric tests cannot be directly applied to price and consumption data from state markets to check the rationality of consumer behavior during the

transition period. Rather, non-parametric tests must be modified to account explicitly for the availability (or lack thereof) of goods in one market and the existence of an alternative market. Without such modifications, the hypothesis of rational consumer behavior could be rejected erroneously.

To illustrate the difficulties with the conventional tests based only on price and consumption data from state shops, assume that the situation in the state shops is depicted in Figure 2.<sup>2</sup> At time period  $i$ , prices are given by  $P_i^s$  (budget line AB). If there are no shortages at state shops in period  $i$ , suppose  $X_k^s$  is chosen while  $X_j^s$  is attainable within the same budget constraint. Further, at time period  $j$ , with the ruling prices  $P_j^s$  (budget line CD), suppose that  $X_j^s$  is chosen while  $X_k^s$  is still attainable within the same budget constraint. This observed behavior is similar to the example, depicted in Figure 1, which violated both WARP and GARP. Accordingly, application of the conventional tests (based on observed prices and quantities from state shops) would reject the hypothesis of rational behavior.

However, suppose that in time period  $j$  there are shortages at the state shops, and  $X_j^s$  is purchased because there are shortages in the state shops, and bundle  $X_k^s$  is not physically available for purchase, although it is attainable within the same budget constraint implied by prices  $P_j$  (e.g.  $P_j^s X_j^s \geq P_j^s X_k^s$ ). By taking into consideration potential purchases from private markets as well, behavior may be rational, despite the results of the conventional non-parametric tests.

The critical question is: “How can we detect the true violations of rationality during the transition period? We must look for the answer in the alternative market

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<sup>2</sup> The superscript  $s$  and  $f$  refer to prices and consumption at state and alternative markets, respectively.

where goods are always available during times of shortage in the state shops, and a part of consumer demand is realized there.

Assume that at time  $j$ , the observed consumption bundle at the state shops is  $X_j^s$ , then  $C_j^j$  is the observed expenditure on the bundle  $X_j^s$  at the ruling state prices  $P_j^s$  :

$$(3) \quad P_{j1}^s X_{j1}^s + P_{j2}^s X_{j2}^s = C_j^j.$$

Now, let us look at the bundle  $X_k^s$  at the same time  $j$ , with the ruling prices  $P_j^s$  at the state shops. The bundle  $X_k^s$  is attainable within the same budget constraint. But, assume the bundle  $X_k^s$  is not *entirely* available at the state shop. Assume that the shortage is given by  $X_{k1}^s - X_{j1}^s$  in Figure 2. This part of the desired consumption bundle can only be purchased in the alternative market at time  $j$ , with the ruling prices  $P_j^f \geq P_j^s$ .

At the alternative market during time period  $j$ , it will cost the consumer  $P_{j1}^f (X_{k1}^s - X_{j1}^s)$  to obtain the physically unavailable part of the consumption bundle  $X_k^s$ , where  $P_{j1}^f$  is the price of good 1 at the alternative market. The physically available part of the bundle  $X_k^s$  can be obtained at the state shops for  $P_{j1}^s X_{j1}^s + P_{j2}^s X_{k2}^s$ , where  $P_{j1}^s, P_{j2}^s$  are the prices of goods 1 and 2 at the state shops, respectively. Hence, the real cost of the consumption bundle  $X_k^s$  is:

$$(4) \quad P_{j1}^s X_{j1}^s + P_{j2}^s X_{k2}^s + P_{j1}^f (X_{k1}^s - X_{j1}^s) = R_k^j.$$

Since prices in the alternative markets are higher than in state shops, we know that:

$$(5) \quad R_k^j > C_j^j$$

By substituting (3) and (4) into (5), and after some simplification we know that:

$$(6) \quad (X_{k1}^s - X_{j1}^s)P_{j1}^f > (X_{j2}^s - X_{k2}^s)P_{j2}^s$$

Given the shortage in the state shops, in this case, the bundle  $X_k^s$  can not be purchased for the budget represented by the line CD in figure 2, and the choice of bundle  $X_j^s$  over  $X_k^s$  purchased from state shops at time period  $j$  should not be a violation of WARP.

In more general terms, consumer behavior during times of shortage in state markets can be modeled easily within Cornes' (1992) treatment of consumer theory with many constraints (pp. 167-186).<sup>3</sup> He shows that the maximization problem in this case leads to the definition of a restricted indirect utility function.

*The modified tests.* This example provides insights into how WARP should be modified in order to check the rationality of consumer behavior during the transition

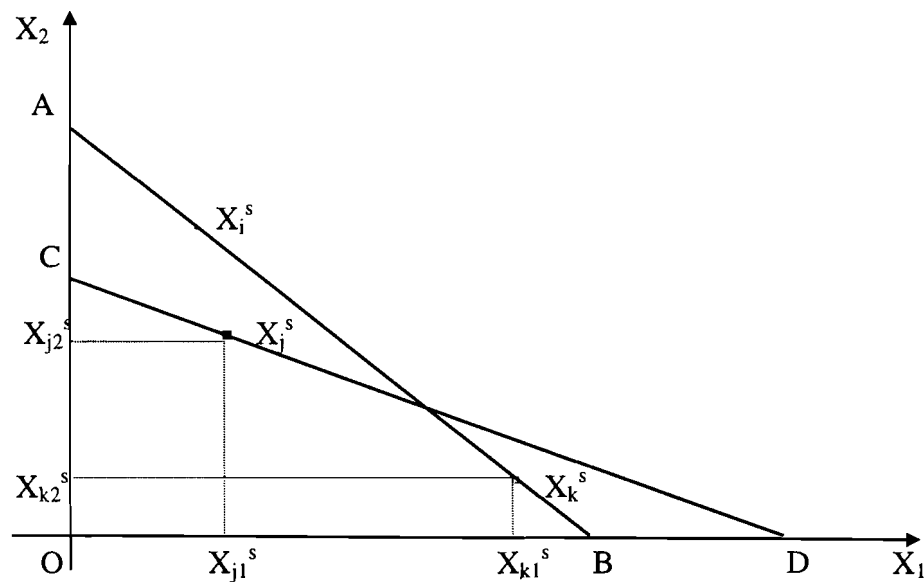


Figure 2: MWARP

<sup>3</sup> The same false violation of WARP when looking only at purchases from state shops could result from an alternative behavioral scenario in which individuals buy some goods from private markets because the difference in prices is lower than the opportunity cost of time waiting in line to purchase goods in state shops. The indirect utility function in this case is also discussed by Cornes (1992, pp. 180-182).

period to a market-oriented economy. In extending non-parametric tests to two markets (state and alternative), we might need data on prices and consumption from state market,  $(P_i^s, X_i^s)$ , as well as data on prices and consumption from the alternative market,  $(P_i^f, X_i^f)$ . Data from state shops, both price and consumption, could be easily obtained. Data on prices from alternative markets could be obtained, too. However, as one might know, it is almost impossible to obtain data on consumption from alternative markets. Thus, an important feature of the modifications of non-parametric tests described below is that they do not require consumption data from the alternative market. To begin, we modify the definitions of the directly revealed preferred:

**Definition 1`** (modified): An observed consumption bundle  $X_i^s$  is directly revealed preferred to a bundle  $X_j^s$ , if  $P_i^s X_i^s \geq P_i^s X_j^s$  and

$$\sum_k (X_{jk}^s - X_{ik}^s)_+ P_{ik}^f + \sum_t (X_{jt}^s - X_{it}^s) \cdot P_{it}^s > 0, \text{ denote } X_i^s R^* X_j^s$$

where  $(X_{jk}^s - X_{ik}^s)_+$  and  $(X_{jt}^s - X_{it}^s)$  are positive and negative elements of the row of  $X_j - X_i$ .

The first constraint in Definition 1` assumes that the entire bundle  $X_j^s$  is physically available for purchase at the state shops during time period  $i$ .<sup>4</sup> On the other hand, the second constraint in Definition 1` takes care of the case when the bundle  $X_j^s$  is *not entirely* physically available for purchase at the state shops during time period  $i$ . The positive elements of  $X_j - X_i$  reflect the shortages, while the absolute values of negative elements reflect additional purchases at state shops as substitutes for goods for which there are shortages.<sup>5</sup>

<sup>4</sup> At time period  $i$ , the bundle  $X_i^s$  has been entirely physically available at the shops because this consumption bundle has been observed.

<sup>5</sup> From equations (3) through (6), it is clear that for the case of two goods, a violation of conventional WARP always satisfies modified WARP, but this is not the case for more than two goods.

**Modified Weak Axiom of Revealed Preference (MWARP):** If  $X_i^s R^s X_j^s$ , then it is not the case when  $X_j^s R^s X_i^s$ .

In terms of expenditures, MWARP implies:

If  $P_i^s X_i^s \geq P_i^s X_j^s$ , then  $P_j^s X_j^s < P_j^s X_i^s$ , or

If  $P_i^s X_i^s \geq P_i^s X_j^s$  and  $P_j^s X_j^s > P_j^s X_i^s$ , then  $\sum_k (X_{jk}^s - X_{ik}^s)_+ P_{ik}^f + \sum_t (X_{jt}^s - X_{it}^s)_- P_{it}^s > 0$   
and  $\sum_k (X_{ik}^s - X_{jk}^s)_- P_{ik}^f + \sum_t (X_{it}^s - X_{jt}^s)_+ P_{it}^s < 0$ ,

where  $(X_{ik}^s - X_{jk}^s)_+$  and  $(X_{it}^s - X_{jt}^s)_-$  are positive and negative elements of the row of  $X_i - X_j$ .

**Definition 2` (modified):** An observed consumption bundle  $X_i^s$  is revealed preferred to a bundle  $X_j$ ,  $X_i P^s X_j$ , if there exist a sequence of bundles  $(X_k^s, X_m^s, \dots, X_r^s)$ , such that  $X_i^s R^s X_k^s, X_k^s R^s X_m^s, \dots, X_r^s R^s X_j^s$ .

**Modified Generalized Axiom of Revealed Preference (MGARP):** If  $X_i$  is revealed preferred to  $X_j$ ,  $X_i P^s X_j$ , then  $X_j$  cannot be revealed preferred to  $X_i$ , not  $X_j P^s X_i$ .

*Conducting the modified tests.* The application of the modified non-parametric tests is straightforward. We begin by conducting the conventional tests of WARP and GARP. The test for WARP is as follows. Let there be  $n=1, \dots, N$  commodities and  $t=1, \dots, T$  time periods. We denote the matrix of prices and consumption as  $P[T, N]$  and  $X[T, N]$ , respectively. The entries  $p_{ij}$  of  $P$  matrix present the price of good  $j$  at time period  $i$ . The entries  $x_{ij}$  of  $X$  matrix present the amount of consumption of good  $j$  at time period  $i$ . Furthermore, a matrix of expenditures  $E[T, T]$  is then constructed with entries  $e_{ij} = \sum_j X_{ij} P_{ij}$ . Each element  $E_{ij}$  represents the expenditure on the consumption bundle  $X_i$  of the  $i$ th period at the prices of the  $j$ th period. The diagonal entries of  $E$  present the actual observed expenditures in each period  $i$ .

Now, we introduce a matrix of ratios  $R[T,T]$  with entries  $r_{ij}=e_{ij}/e_{jj}$ . If element  $r_{ij} \leq 1$ , then  $X_j$  is directly revealed preferred to  $X_i$ , i.e. bundle  $X_i$  was attainable at the prices of the period  $j$ , but bundle  $X_j$  was chosen. If  $r_{ij} \leq 1$  and  $r_{ji} \leq 1$ , then we have a violation of WARP. For further purposes, let us define matrix  $W[T,T]$  that shows violations of WARP. The elements of  $W$  will be 1 if there is a violation of WARP and 0 otherwise, so

$$(7) \quad w_{ij} = \begin{cases} 1, & \text{if } r_{ij} \leq 1 \text{ and } r_{ji} \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

In tests for GARP, we check the transitivity with a matrix  $M[T,T]$  with entries:

$$(8) \quad m_{ij} = \begin{cases} 1, & \text{if } r_{ij} \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Now, the algorithm suggested in Varian (1982) can be applied. We create a matrix  $MT[T,T]$  with entries:

$$(9) \quad mt_{ij} = \begin{cases} 1, & \text{if } m_{ik}=1 \text{ and } m_{kj}=1 \\ 0, & \text{otherwise.} \end{cases}$$

GARP is violated if  $mt_{ij}=1$  and  $mt_{ji}=1$ . For further purposes let us define matrix  $G[T,T]$  that shows violations of GARP. The elements of  $G$  will be 1 if there is a violation of GARP and 0 otherwise, so:

$$(10) \quad g_{ij} = \begin{cases} 1, & \text{if } mt_{ij}=1 \text{ and } mt_{ji}=1 \\ 0, & \text{otherwise.} \end{cases}$$

The GAMS codes for WARP and GARP are given in Appendix A.

Having obtained the matrices of violations of WARP and GARP,  $W[T,T]$  and  $G[T,T]$ , we proceed to extend non-parametric tests of consumer behavior to two, state and



alternative, markets. Examples from Figure 2 and definitions 1` and 2` are the basis for the modification.

The following algorithm explains how MWARP and MGARP work.

*Algorithm for MWARP (MGARP):*

- A. Detect a violation of the conventional WARP (GARP),  $w_{ij}(g_{ij})=1$  in the matrix  $W(G)$ .
- B. Make a row  $F_{ij}$  by subtracting consumption row of time period  $j$ , from the consumption row of time period  $i$ :

$$F_{ij}[1,T] = X_i[1,T] - X_j[1,T]$$

- C. Form a new row of prices,  $H_{ij}[1,T]$ .

If the  $k$ th element of  $F_{ij}$  is positive, then the  $k$ th element of the new price row,  $h_k$ , is equal to the element  $P_{jk}^f$  of the matrix of alternative market prices, otherwise  $h_k$  is equal to  $P_{jk}^s$  element of the matrix of state shops prices.

- D. Calculate  $M_{ij}$  and  $M_{ji}$  as follows:

$$M_{ij} = F_{ij} H_{ij}^T$$

- E. If  $M_{ij} > 0$  and  $M_{ji} > 0$ , then there is no violation of the modified WARP(GARP), although the conventional WARP(GARP) is violated.

The elements of matrices reflecting violations of MWARP,  $MW[T,T]$ , and MGARP,  $GW[T,T]$ , are given by:

$$(11) \quad mw_{ij} = \begin{cases} 1, & \text{if } w_{ij}=1 \text{ and } M_{ji} < 0 \text{ and } M_{ij} < 0 \\ 0, & \text{otherwise, and} \end{cases}$$

$$(12) \quad mg_{ij} = \begin{cases} 1, & \text{if } g_{ij}=1 \text{ and } M_{ji} < 0 \text{ and } M_{ij} < 0 \\ 0, & \text{otherwise.} \end{cases}$$

The matrices  $MW[T,T]$  and  $MG[T,T]$  assign 1 if there is a violation of modified WARP or GARP, and 0 otherwise.

### *An Application*

To apply these modified tests using data from the Russian Federation, we begin by conducting the conventional non-parametric tests, WARP and GARP. We then find violation cases of WARP and GARP, and subject them to the modified WARP and GARP to test the true violations of WARP and GARP during the transition period.

*The Data.* The data used in this illustration of the new non-parametric methods were obtained from the Statistical Committee of the Russian Federation. They are aggregate prices and consumption data for: meat, milk, eggs, and potatoes.<sup>6</sup> These annual data are from five cities (Moscow, Leningrad, Arkhangelsk, Krasnodar, Khabarovsk) for 3 years (1992, 1993, 1994). Per capita consumption at the state shops, for these 15 data points, are given in Table 1, while state market prices for these goods are in Table 2.

Fortunately, we were also able to obtain data on the prices at the alternative market and they are presented in Table 3. As might be expected, these prices are from two or three times higher than those at state shops. Although some data points in Table 3 are missing, we do make use of the information available in conducting the modified non-parametric tests.

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<sup>6</sup> Since the data cover only these four food items and not other budget items, standard separability assumptions must be made to apply the modified tests. These goods must be weakly separable from the other goods not represented, and various types of meat must be homothetically separable from other food so that one can define a meat aggregate (Driscoll and McGuirk, 1997; Diewert, 1976; and Blackorby, Primont and Russell, 1978). Clearly, the validity of these assumptions is an empirical question that cannot be resolved with the limited availability of data. The data are sufficient, however, to illustrate the new non-parametric methods.

Table 1: Yearly Per Capita Consumption

Cities	Year		Meat (kg)	Milk (liter)	Eggs (number)	Potatoes (kg)
			G1	G2	G3	G4
Moscow	1992	I1	60	300	279	83
Leningrad	1992	I2	56	281	424	135
Arkhangel'sk	1992	I3	48	200	232	77
Krasnodar	1992	I4	58	207	242	79
Khabarovsk	1992	I5	47	202	206	51
Moscow	1993	I6	58	316	280	84
Leningrad	1993	I7	54	281	368	135
Arkhangel'sk	1993	I8	46	218	217	72
Krasnodar	1993	I9	55	206	206	92
Khabarovsk	1993	I10	45	171	213	52
Moscow	1994	I11	59	281	272	84
Leningrad	1994	I12	35	276	320	135
Arkhangel'sk	1994	I13	44	206	200	77
Krasnodar	1994	I14	53	244	208	85
Khabarovsk	1994	I15	41	171	196	56

Source: Statistical Committee of the Russian Federation

Table 2: Prices per unit in Roubles at the State Shops

Cities	Year		Meat (kg)	Milk (liter)	Eggs (number)	Potatoes (kg)
			G1	G2	G3	G4
Moscow	1992	I1	97.67	10.09	4.12	15.27
Leningrad	1992	I2	106.82	14.16	4.55	19.56
Arkhangel'sk	1992	I3	114.13	18.55	3.79	20.57
Krasnodar	1992	I4	110.57	16.95	3.45	15.86
Khabarovsk	1992	I5	121.90	23.90	6.30	21.54
Moscow	1993	I6	1229.92	154.58	33.23	127.62
Leningrad	1993	I7	1166.28	155.64	31.09	131.97
Arkhangel'sk	1993	I8	1207.61	188.00	31.010	159.74
Krasnodar	1993	I9	1080.95	168.36	32.23	104.82
Khabarovsk	1993	I10	1480.43	284.54	36.91	194.68
Moscow	1994	I11	3627.86	718.49	130.06	515.06
Leningrad	1994	I12	3400.19	761.64	112.25	547.03
Arkhangel'sk	1994	I13	3815.13	890.38	144.97	784.35
Krasnodar	1994	I14	2424.38	646.73	115.44	546.66
Khabarovsk	1994	I15	4696.19	1486.98	208.78	1013.97

Source: Statistical Committee of the Russian Federation

Table 3: Prices per unit in Roubles at the Alternative Market

Cities	Year		Meat (kg)	Milk (liter)	Eggs (number)	Potatoes (kg)
			G1	G2	G3	G4
Moscow	1992	I1	366.3	74.3	6.8	31.4
Leningrad	1992	I2	283.7	NA	10.3	25.7
Arkhangel'sk	1992	I3	168.9	NA	NA	40.0
Krasnodar	1992	I4	165.8	24.8	3.8	20.3
Khabarovsk	1992	I5	250.0	62.4	6.3	27.9
Moscow	1993	I6	4,320.4	636.8	61.3	231.3
Leningrad	1993	I7	2,950.8	NA	66.9	151.7
Arkhangel'sk	1993	I8	1,880.9	NA	NA	274.9
Krasnodar	1993	I9	1,588.7	179.0	35.5	127.6
Khabarovsk	1993	I10	2,899.9	763.4	51.3	250.3
Moscow	1994	I11	10,110.6	2,983.8	213.5	834.1
Leningrad	1994	I12	6,555.4	75	179.1	554.2
Arkhangel'sk	1994	I13	6,032.6	NA	NA	1,014.1
Krasnodar	1994	I14	4,036.8	635.2	125.9	566.3
Khabarovsk	1994	I15	7,928.0	3,662.6	422.4	1,207.1

Source: Statistical Committee of the Russian Federation.

*Test Results for WARP, GARP, MWARP, and MGARP.* By executing the GAMS codes from Appendix A, we obtain the matrices of violations of WARP and GARP. The results are given in Appendix B.

If the entry  $w_{ij}$  of  $W[15,15]$  matrix is 1, then we have a violation of the conventional WARP, and this violation involves the  $i$ th and  $j$ th time periods. Accordingly, the violation points are: (I6,I1), (I4,I12), (I9,I12), (I1,I6), (I12,I4), (I12,I9). Having obtained the violations of the conventional WARP, the modified WARP is applied to detect the true violations of WARP. For this purpose, we need the prices at the alternative market and, fortunately, there are no missing data in Table 3 for the cases of interest, which are I1, I6, I4, I9, and I12.

To check MWARP, we start with the case of (I1,I6). Below, we present how MWARP is checked for this case. We apply the *MWARP algorithm* from above in a straightforward fashion:

- A. Detect a violation case of the conventional WARP,  $w_{16}=w_{61} = 1$  in the matrix  $W$ .
- B. Make a row  $F_{1,6}$  as follows: From Table 1, subtract consumption row of time period I6 from the consumption row of time period I1:

$$F_{1,6}[1,4] = X_1[1,4]-X_6[1,4] = (2,-16,-1,-1)$$

- C. Then, we form a new row of prices,  $H_{1,6}[1,4]$ : if the  $k$ th element of  $F_{1,6}$  is positive, then the  $k$ th element of the new price row,  $h_k$ , is equal to the element (I6,GK) of the matrix of alternative market prices (Table 3), otherwise  $h_k$  is equal to (I6,GK) element of the matrix of state shops prices (Table 2).

$$H_{1,6} = (4320.4, 155.64, 310.91, 131.97)$$

- D. Calculate  $M_{1,6}$  and  $M_{6,1}$

$$M_{1,6} = (2,-16,-1,-1) \cdot (4320.4, 154.58, 33.225, 127.62)^T = 6006.74$$

$$M_{6,1} = 1031.99$$

- E. As  $M_{1,6} > 0$  and  $M_{6,1} > 0$ , the case of  $w_{16}$  and  $w_{61}$  is not a true violation; MWARP is satisfied, although the conventional WARP is violated.

Applying the same algorithm to calculate  $M_{4,12}$   $M_{9,12}$   $M_{12,4}$   $M_{12,9}$ , we have:

$$M_{4,12} = 58832 \text{ and } M_{12,4} = 599$$

$$M_{9,12} = 41475 \text{ and } M_{12,9} = 438$$

The conventional WARP produced 6 cases of violation for consumer behavior. However, MWARP applied to these *pseudo* violations shows no cases of true violation.

Now we must turn our attention to the cases where GARP is violated. From Appendix B, these are:

(I6,I1), (I4,I12), (I9,I12), (I1,I6), (I12,I4), (I12,I9), (I4,I9) and (I9,I4).

The list of GARP violations must include all the violations of WARP; there are two additional violations, namely, (I4,I9) and (I9,I4).

To conduct MGARP, we must apply the modification to the list of violations of GARP.  $M_{1,6}$  and  $M_{6,1}$ ,  $M_{4,12}$  and  $M_{9,12}$ ,  $M_{12,4}$  and  $M_{12,9}$  are calculated for MWARP and they are the same for MGARP. The only new point to which we must apply MGARP is (I4,I9) and (I9,I4). For this purpose  $M_{4,9}$  and  $M_{9,4}$  must be calculated from the MGARP algorithm above:

- A. Detect a violation case of the conventional GARP,  $g_{49}=g_{94} = 1$  in the matrix G.
- B. Make a row  $F_{4,9}$ : From Table 1, subtract consumption row of time period I9 from the consumption row of time period I4:

$$F_{4,9}[1,4] = X_4[1,4]-X_9[1,4] = (3,1,36,-13)$$

- C. Then, form a new row of prices,  $H_{4,9}[1,4]$ :

if the  $k$ th element of  $F_{4,9}$  is positive, then the  $k$ th element of the new price row,  $h_k$ , is equal to the element (I9,GK) of the matrix of alternative market prices (Table 3), otherwise  $h_k$  is equal to (I9,GK) element of the matrix of state shops prices (Table 2).

$$H_{4,9} = (1588, 179, 35, 105)$$

- D. Calculate  $M_{4,9}$  and  $M_{9,4}$

$$M_{4,9} = (3,1,36,-13) (1588, 179, 35, 105)^T = 4838$$

$$M_{9,4} = (-3,-1,-36,13) (110.5, 16.95, 3.446, 20.3)^T = -264$$

E. Since  $M_{4,9} > 0$ , there is no true violation at (I4,I9). However,  $M_{9,4} = -264 < 0$ , which indicates that the violation at the point (I9,I4) is a true violation of GARP.

*A summary.* In terms of conventional WARP, only 6 violations (from a possible 105) were found, involving 5 observations. The modified WARP applied to the same data set shows no cases of *true* violations. All six violations of the conventional WARP were *pseudo* violations, i.e. the case described in the Figure 2.

With respect to the conventional GARP, there were 8 observations involved in the violations, however, no additional data points were involved. The modified GARP applied to the same data set shows only one true violation, the other seven were pseudo violations. The only violation sample observation is (I9,I4) and the visual inspection shows that this violation represents bundles such as  $X_j$  and  $X_k$  from Figure 2, but both bundles are very close to the intersection point of the budget lines.

### ***Conclusions***

In this paper, we modify the non-parametric tests of the axioms of revealed preference to examine consumer behavior and the notion of consumer rationality in transition economies by recognizing explicitly the existence of both a state market and an emerging private market. To reflect the unique characteristics of transition economies, shortages are allowed in the state market, while not in the private market.

Because of the way the modified tests are constructed, data for prices in both the state and private markets are needed, as are data on purchases from state markets. Through some convenient algebraic substitutions, the tests do not require data on actual quantities purchased in the private markets. This feature of the modified tests is

especially important, given the near impossibility of gaining access to data on sales or purchases from private markets in transition economies.

The method is demonstrated using aggregate consumption data for four important food items in five cities of the former Soviet Union. Although data are limited, we demonstrate that if observed purchases from state shops are due to shortages in the supply of some goods, the conventional non-parametric tests can lead to an erroneous rejection of the rationality hypothesis of consumer behavior in transition economies. Clearly, if consumption data from private markets were available, one could interpret the results of these modified tests with greater confidence.

Although the focus here is on dealing with the effects of shortages in state markets, there are other situations for which similar modified non-parametric tests might shed light on the rationality of consumer behavior. In transition economies, for example, one could account for differential product quality or the implicit value for time if there are long queries required for state shop purchases. In developed economies, on the other hand, many two-career households have high incomes, but face severe time constraints. To accommodate busy schedules, these households are increasing their reliance on alternative markets. They have purchased more meals outside the home and have increased dramatically their purchases through direct TV marketing, on-line purchases via the Internet, catalog shopping, and purchases through factory outlet stores. In future years, an understanding of consumer behavior will require explicit recognition of these alternatives to conventional shopping.



## ***Bibliography***

- Afriat H.S. The Construction of Utility Functions from Expenditure Data. *International Economic Review*, 8 (1967), N1, p.67-77.
- Blackorby, C., D. Primont and R. Russell. *Duality, Separability, and Functional Structure: Theory and Economic Applications*. New York: North-Holland, 1978.
- Clower R.W. *The Keynesian Counterrevolution: A Theoretical Appraisal* in F.P.R. Brechling. Macmillan Press, 1965.
- Cornes, R. *Duality and Modern Economics*. Cambridge: Cambridge University Press, 1992.
- Davis C., and W. Charemza *Models of Disequilibrium and Shortage in CPEs*. London: Chapman and Hill Press, 1989.
- Diewert, W.E. "Exact and Superlative Index Numbers", *J. Econometrics* 4(May 1976):115-45.
- Driscoll P., and A. McGuirk. "Dietary Bounds and Unshackled Demand Specifications" *American Journal of Agricultural Economics* 79(May 1997):566-572.
- Kornai J. *Anti-Equilibrium*. North Holland, Amsterdam, 1971.
- Kornai J. *Economics of Shortage*. North Holland, Amsterdam, 1980.
- Mekhtiev, A. An Analysis of the Evolution of Consumer Behavior in the Emerging Markets: Transition to Market Economies", M.S. Department of Agricultural, Resource, and Managerial Economics, Cornell University, August 1997.
- Shananin A.A. Neparametricheskie Metody Analiza Strukturi Potrebitel' Skogo Sprosa. *Mathematical Modeling* 5(1993, N.9):3-17.
- Shananin, A.A. *Agregirovanie Konechnikh Produktov i Problema Integriruемости Funktsiy Sprosa*. Moscow, VTS Academy of Science of USSR, 1990.
- Varian H. The Non-parametric Approach to Demand Analysis. *Econometrica*, 50(1982):945-73.
- Varian H. Non-parametric Tests of Consumer Behavior. *The Review of Economic Studies*, L(1983):99-110.

## Appendix A

\*This program tests the WARP and GARP hypotheses.

### SETS

I observations /I1\*I15/

G goods /Meat, Milk, Eggs, Potatoes/;

ALIAS (I,J,K);

### SCALAR

PjXj

PjXi;

### PARAMETER

R(I,J) ratios of expenditures

M(I,J) Varian's Mmatrix

MT(I,J) Varian's MTmatrix

WARP(I,J) matrix of WARP results

GARP(I,J) matrix of GARP results;

TABLE P(I,G) observed prices

	Meat	Milk	Eggs	Potatoes
I1	97.67	10.09	4.115	15.27
I2	106.82	14.16	4.554	19.56
I3	114.13	18.55	3.790	20.57
I4	110.57	16.95	3.446	15.86
I5	121.90	23.90	6.309	21.54
I6	1229.92	154.58	33.225	127.62
I7	1166.28	155.64	31.091	131.97
I8	1207.61	188.00	31.099	159.74
I9	1080.95	168.36	32.228	104.82
I10	1480.43	284.54	36.93	194.68
I11	3627.86	718.49	130.058	515.06
I12	3400.19	761.64	112.245	547.03
I13	3815.13	890.38	144.972	784.35
I14	2424.38	646.73	115.443	546.66
I15	4696.19	1486.98	208.776	1013.97;

TABLE X(I,G) consumption

	Meat	Milk	Eggs	Potatoes
I1	60	300	279	83
I2	56	281	424	135
I3	48	200	232	77
I4	58	207	242	79
I5	50	191	245	157
I6	58	316	280	84
I7	54	281	368	135
I8	46	218	217	72
I9	55	206	206	129
I10	47	191	234	52
I11	59	281	272	84
I12	35	276	320	135
I13	44	206	200	77
I14	53	244	208	85
I15	47	173	189	87;

\* Computation of expenditure ratios and Varian's M matrix

```

LOOP(J,
  PjXj = SUM(G,P(J,G)*X(J,G));
  LOOP(I,
    PjXi = SUM(G,P(J,G)*X(I,G));
    R(I,J) = PjXi/PjXj;
    M(I,J) = 1$(R(I,J) LE 1 AND R(J,I) LE 1);
  ));

LOOP(K,
  LOOP(I,
    LOOP(J,
      MT(I,J) $ (M(I,K)=1 AND M(K,J)=1) = 1;
    ));

LOOP(I,
  LOOP(J,
    WARP(I,J) = 1 $(M(I,J)=1 AND M(J,I) EQ 1);
    GARP(I,J) = 1 $(MT(I,J)=1 AND MT(J,I) EQ 1);
  ));

DISPLAY  WARP, GARP;

```

## Appendix B

Table B.1. Violations of WARP

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10	I11	I12	I13	I14	I15
I1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
I2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I4	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
I5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I9	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
I10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I12	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0
I13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table B.2. Violations of GARP

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10	I11	I12	I13	I14	I15
I1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
I2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I4	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0
I5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I9	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0
I10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I12	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0
I13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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